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Robust Cascade Control of Gas Turbines in the Presence of Sensor Faults

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Dedication

To my parents:
*To my father, **Mabrouk** a strong and kind man who taught me a lot just by being who he is.*
*And to my mother, **Aicha** the heart of our home, whose love and support have always been with me.*

No words can express the depth of my gratitude.
May God protect you both.

*To my brother **Yazen,** my closest supporter and friend,*

*And to my sisters **Amina** and **Loubna** your advice, care, and the way you've helped me grow will always stay with me. I've learned so much from you.*

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I'm proud to have walked this path with you.

Hani Malem

Dedicace

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Abstract :

This thesis presents a robust control strategy for gas turbine systems using a cascade control structure designed to withstand sensor faults and time delays. The study focuses on the combustion chamber subsystem, where temperature regulation is critical to system safety and efficiency. A mathematical model with two interconnected loops : temperature and gas flow is developed to simulate sensor failure and delay scenarios. To ensure system stability under these conditions, the Lyapunov-Krasovskii functional method combined with Linear Matrix Inequalities (LMIs) is applied to derive sufficient stability conditions. The proposed approach allows for the synthesis of a state-feedback controller capable of maintaining performance despite sensor inaccuracies and communication-induced delays. Simulation results confirm the effectiveness of the control design in stabilizing the system and mitigating the effects of faults.

Keywords: Gas turbine, sensor fault, cascade control, Lyapunov-Krasovskii, LMI, time-delay system, fault-tolerant control.

ملخص :

تقدم هذه الأطروحة استراتيجية تحكم قوية لأنظمة التوربينات الغازية باستخدام هيكل تحكم متتالي مصمم لتحمل أعطال المستشعرات والتأخيرات الزمنية. تركز الدراسة على النظام الفرعي لغرفة الاحتراق، حيث يعد تنظيم درجة الحرارة أمرًا بالغ الأهمية لسلامة النظام وكفاءته. تم تطوير نموذج رياضي به حلقتان مترابطتان لدرجة الحرارة وتدفق الغاز لمحاكاة سيناريوهات فشل المستشعرات والتأخير. لضمان استقرار النظام في ظل هذه الظروف، تم تطبيق طريقة Krasovskii Lyapunov- الوظيفية جنبًا إلى جنب مع متباينات المصفوفة الخطية (LMIs) لاستنباط شروط الاستقرار الكافية. تسمح الطريقة المقترحة بتوليف وحدة تحكم تغذية مرتدة قادرة على الحفاظ على الأداء على الرغم من عدم دقة أجهزة الاستشعار والتأخيرات الناتجة عن الاتصالات. تؤكد نتائج المحاكاة فعالية تصميم التحكم في استقرار النظام وتخفيف آثار الأعطال. **الكلمات المفتاحية :** توربين غازي، عطل في المستشعر، تحكم متتالي، Lyapunov-Krasovskii، LMI، نظام تأخير زمني، تحكم متسامح مع الأعطال.

Résumé :

Cette thèse présente une stratégie de contrôle robuste pour les systèmes à turbine à gaz utilisant une structure de contrôle en cascade conçue pour résister aux défaillances des capteurs et aux retards. L'étude se concentre sur le sous-système de la chambre de combustion, où la régulation de la température est essentielle à la sécurité et à l'efficacité du système. Un modèle mathématique avec deux boucles interconnectées, température et débit de gaz, est développé pour simuler des scénarios de défaillance et de retard des capteurs. Afin de garantir la stabilité du système dans ces conditions, la méthode fonctionnelle de Lyapunov-Krasovskii combinée aux inégalités matricielles linéaires (LMI) est appliquée pour dériver des conditions de stabilité suffisantes. L'approche proposée permet la synthèse d'un contrôleur à rétroaction d'état capable de maintenir les performances malgré les imprécisions des capteurs et les retards induits par la communication. Les résultats de la simulation confirment l'efficacité de la conception du contrôle pour stabiliser le système et atténuer les effets des défaillances.

Mots-clés : turbine à gaz, défaillance de capteur, contrôle en cascade, Lyapunov-Krasovskii, LMI, système à retard, contrôle tolérant aux défaillances.

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List of Abbreviations

Symbols	Designations
LMI	Linear Matrix Inequality
LKF	Lyapunov-Krasovskii Functional

Chapter 1:

State of the art on the basic principles of fault diagnosis

1.1 Introduction

Gas turbines are key parts of modern energy production, where controlled and reliable combustion is required to ensure system performance and security. The combustion chamber requires a precise control of temperature and fuel flow to provide optimal operation. The realistic systems suffer from inherent problems such as sensor faults and time delays, which destabilize the control loops and negatively influence performance. In particular, failures in temperature sensors can lead to spurious feedback, creating system instability and inappropriate control responses. This dissertation addresses the aformentioned problems by introducing an aggressive cascade control approach for gas turbines that is resilient in the presence of such sensor failures. The proposed method adopts the Lyapunov-Krasovskii functional approach and Linear Matrix Inequalities (LMIs) to ensure stability despite uncertainties and delays, providing a robust solution for real-time gas turbine control systems.

1.2 Motivation

During our academic research and practical study of control systems, we observed how real-world systems, and in particular gas turbines, are highly vulnerable to sensor faults and time delays. These imperfections may result in instability, inefficiency, and even operational risks. These issues collectively motivated us to investigate more robust and fault-tolerant control techniques. We were specifically interested in the cascade control structure due to its industrial relevance and applicability in controlling multiloop systems. The current study's main emphasis is on the Lyapunov-Krasovskii method due to its effectiveness in resolving time-delay problems and ensuring system stability.

1.3 Problem Statement

The system in question contains two coupled control loops: a temperature loop and a gas flow loop. The temperature loop is measuring the temperature of the combustion chamber T and is regulating the gas valve to manipulate the gas flow G. The control strategy is that the gas flow is reduced by a rise in temperature and vice versa in order to maintain the combustion chamber at a constant temperature. Failure of temperature sensors, though, results in measurement errors in temperature, which can destabilize the system. Delays in measurement also complicate the dynamics, necessitating an analysis of stability that will account for faults in addition to delays.

1.4 Objectives and Methodology

The main goal of this work involves creating a theoretical framework to study combustion chamber system stability when temperature sensors fail and measurement delays occur. The Lyapunov-Krasovskii method serves as the established method for analyzing stability in time-delay systems [5]. The method allows building a Lyapunov functional which guarantees stability during delays and bounded disturbances including the sensor fault [4]. The study mainly aims to determine a stabilizing control gain K which will provide robust performance. The methodology involves: Modeling the system dynamics with the calculated matrices. Analyzing the impact of the temperature sensor fault. The Lyapunov-Krasovskii method enables us to obtain stability conditions for the system [5]. The stability conditions are expressed as LMIs [12]. A numerical optimization method will be used to determine the control gain K, the theoretical aspects and boundaries of the research will be discussed. ‘

1.5 Theoretical Contribution to System Dynamics

The presence of fault and delay affects the transient and steady-state response of the system [14]. The absence of stabilizing controller, may make the fault causing divergence of temperature from setpoint, leading to repeated oscillations or unbounded growth of temperature error. The delay also aggravates this by introducing a disparity between the current state and control action, which has the potential to destabilize a stable system [11]. Stability analysis throughout the following section is aimed at resolving such problems by determining conditions where the system remains stable despite fault and delay.

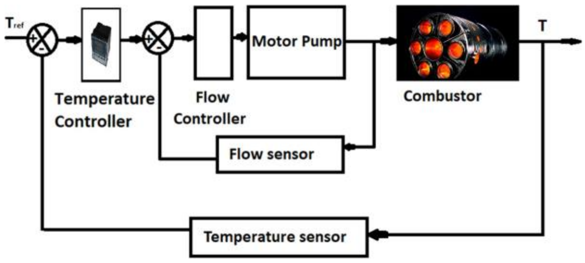

Figure 1.1: Control scheme of the overall system.

1.6 Overview of Combustion Chamber Control

The combustion chamber serves as a critical component in industrial applications including power station furnaces and engines because it requires temperature control for both safety and performance efficiency. The majority of combustion processes require controlled fuel-oxidizer reactions which produce heat through oxygen involvement. The system requires sufficient temperature control to maintain chamber temperatures at specific levels because high temperatures damage equipment and insufficient heat reduces operational efficiency [1]. Furthermore, the system components achieve extended lifespan through optimal fuel consumption and emission reduction becomes possible through proper control. In fact, the combustion chamber control systems use feedback mechanisms to adjust two essential variables which include chamber temperature and fuel (or gas) flow rate. Temperature loop is used to sense chamber temperature to control flow of gas by a valve to achieve set-point. Gas flow loop affects combustion processes whose effect is on the level of temperatures. Interconnectedness of the systems creates a coupled system where disturbance to one of the loops may spread to the other and could create instability faults, especially in the sensor is one of the most prevalent problems in such systems [15]. The temperature sensors which provides the feedback for control can fail with respect to biases, drifts, or noises and begin giving erroneous measurements. The errors in the measurement will drive the control loop unstable, and the system will deviate from the operating point. Moreover, measurement or actuation delays — resulting from sensor response times, communication delays, or mechanical delays in the valve — can extend the influence of faults, making stability analysis difficult [14].

1.7 Type of Faults in Temperature Sensor

1.7.1 Environmental Factors

The failure of temperature sensors occurs because of severe environmental factors which include both harsh temperatures and high humidity and corrosive substances. The sensor components experience damage when exposed to extreme heat or cold temperatures which leads to performance drift or system malfunction. The sensor contacts develop corrosion when exposed to corrosive chemicals and high humidity levels which results in inaccurate measurements.

1.7.2 Electrical Issues

The temperature sensor failure results from a variety of factors including power surge, voltage spikes and incorrect wiring. The circuitry of the sensor can be damaged because of power fluctuations and incorrect levels of voltage. These issues may also make the circuitry less effective or cause inaccurate readings

1.7.3 Physical Damage

the Physical damage to the temperature sensor will cause it to fail when it experiences impact or vibration or improper handling. The internal components of the sensor get damaged when the device experiences rough installation handling or accidental bumps or excessive mechanical stress which results in performance degradation.

1.7.4 Aging and Wear

Temperature sensors can decline or fail as a result of aging and wear over a span of time. The sensor functionality can be affected by the continuous exposure to temperature cycling and thermal stress and operational conditions. The functionality of the sensor can be affected by aging parts like deteriorated cable or worn-out sensing components which may cause drift or unusual readings [13].

1.8 Effects of Temperature Sensor Failure

1.8.1 Inaccurate Temperature Readings

The primary result of temperature sensor failure leads to incorrect temperature measurement. A faulty sensor generates both incorrect readings and unpredictable measurements, which negatively impact both process control and product quality and safety.

1.8.2 Process Disruptions

The breakdown of temperature sensors creates operational disruptions to manufacturing processes and system control systems which require exact temperature control. The pharmaceutical industry together with food processing and chemical manufacturing require exact temperature control to preserve product quality while fulfilling regulatory standards. A malfunctioning temperature sensor results in process inefficiencies and reduced product quality and production delays.

1.8.3 Safety Risks

The failure of temperature sensors in safety-critical applications, including industrial ovens and HVAC systems and electrical equipment creates safety hazards. The potential of equipment overheating and fire hazards as well as personal injuries increases when abnormal temperature levels or malfunctions remain undetected.

1.8.4 Equipment Damage

Equipment damage occurs when temperature sensors fail. Equipment damage can occur when temperature sensors fail in critical systems such as cooling systems, engines or power distribution units because it leads to inadequate cooling and excessive heating and improper shutdown functions [13].

1.9 Control Strategies for Mitigating Temperature Sensor Faults

1.9.1 Classical PID Controller

A classical PID controller represents an industrial control structure that combines three control actions: P (Proportional), I (Integral), and D (Derivative) to reduce the error between the setpoint and the measured process variable in a closed-loop system. The proportional term corrects the current error, the integral term eliminates steady-state error by adding past errors, and the derivative term looks ahead to predict future errors by responding to the rate of change of the error, offering solid and good control in a broad variety of applications [10].

1.9.2 Sliding Mode Controller

Sliding mode control is a nonlinear control method widely used in engineering design. This control method shows good robustness by designing appropriate sliding surface, ensuring satisfactory response performance in the presence of model parameter deviation [8].

1.9.3 State Feed-back Controller

When a system is affected by faults—such as sensor drift, bias, or partial failure—the accuracy of the measured output degrades, which can directly impact the control performance. A State Feedback Controller can be effectively used to control such a faulty system by relying on the internal state of the system rather than directly on the faulty measurements. In this context, a state observer or estimator becomes essential to reconstruct the system states using the available (possibly corrupted) outputs.

The integration of control techniques, such as state feedback or PID control—with Lyapunov-based methods plays a fundamental role in the design and analysis of robust and fault-tolerant control systems. Lyapunov theory provides a rigorous mathematical framework to analyze the stability of dynamic systems, especially under uncertainties or faults. In the case of state feedback control, the gain matrix K is often designed not only to achieve desired pole placement or optimality (as in LQR) but also to guarantee that the closed-loop system satisfies a Lyapunov stability condition.

1.10 Construction of Lyapunov function

A. Lyapunov method [6]: The Lyapunov stability method can be intuitively understood in terms of energy concepts. In this framework, the Lyapunov function $V(x)$ is interpreted as a generalized energy stored in the system. For a system to be stable, this energy should not increase over time. Specifically, if $V(x)$ is positive definite, i.e., $V(x) > 0$ for all $x \neq 0$ $V(0) = 0$, it represents the "potential energy" of the system. The time derivative $\dot{V}(x)$ describes the rate at which this energy changes along the system trajectories.

B. Lyapunov Function Construction [6]:

- Consider a nonlinear system: $x^{\cdot} = f(x)$
- Construct a Lyapunov function candidate: $V(x) = x^T Px = f(x)^T P f(x)$ where P is a positive definite matrix.
- Then analyse: $\dot{V}(x) = \frac{d}{dt}^{\cdot} f(x)^T P f(x)^{\cdot}$
- If $\dot{V}(x) \leq 0$ and $V(x) > 0$, then the system is **stable**.

1.10.1 Lyapunov-Krasovskii advantages

The The Lyapunov-Krasovskii method demonstrates superior advantages through its ability to provide a strong adaptable system for time-delay analysis that covers numerous engineering fields. The Lyapunov-Krasovskii method demonstrates exceptional robustness through its capability to handle both steady and variable time delays and system uncertainties thereby providing essential functionality in aerospace control systems with satellite delays and automotive applications like anti-lock braking systems with sensor delays. The computational efficiency of this method enables stability assessments to convert into linear matrix inequality format which allows engineers to use MATLAB software for simplified and quick analysis of complex systems. The Lyapunov-Krasovskii method provides adaptable solutions for multiple system types including linear and nonlinear systems and distributed delays and delay-dependent and delay-independent criteria for optimal results. The method enables controller development for delay compensation which enhances real-time system performance and reliability in applications such as chemical plant temperature stabilization and telecommunications network data delay management. The method proves useful in practice through its ability to maintain stability in systems with slow sensors like distillation columns in refineries as well as internet-based robotic control systems with latency. The method presents challenges through its delay-independent analysis conservatism but refined delay-dependent methods solve these issues. This method's advantages resemble the process of handling past experiences with delayed emotional responses because it uses a flexible approach to achieve balance and clarity in the present just like the method converts difficulties into stable opportunities [7].

1.11 Linear matrix inequality (LMI)

The use of LMI (linear matrix inequality) techniques allows for more flexibility in the design of dynamic linear systems than the techniques that minimize a scalar functional for optimization. For linear state space models, multiple goals (performance bounds) can be characterized in terms of LMIs, and these can serve as the basis for controller optimization via finite-dimensional convex feasibility problems. This entry describes LMI formulations of various standard control problems including dynamic feedback stabilization, covariance control. The formulation of the feasibility constraints as LMIs is shown to be a convex problem for the integration of control and information architecture design (i.e., adding sensor/actuator precision selection to the control problem) [2].

1.12 Sensor Precision and Stability

The precision and stability of sensors determine their ability to measure temperature changes in control loops that contain valves and gas flow systems which affects gas flow adjustments. The precise sensor detects small temperature changes above 150 degrees Celsius to trigger valve gas flow reduction and the stable sensor delivers reliable readings through time and vibration and temperature changes. The system would experience incorrect temperature readings of 160 degrees Celsius instead of 150 degrees Celsius due to poor precision which would result in excessive gas flow reduction. Sensor readings instability would generate unpredictable measurements which would disrupt the control process. The system achieves precise temperature control through high-quality sensors that undergo regular calibration to avoid both temperature overshoots and drops. The memory narrative shows how precise and consistent past recollections work like sensor precision to guide emotional stability in current situations [3].

1.13 Gas Flow Optimization Strategies (Lyapunov-Krasovskii)

The best gas flow optimization strategy involves Lyapunov-Krasovskii because it addresses delays in the control loop between the temperature sensor and valve. The gas flow optimization strategies adjust the gas flow through a control loop with temperature sensor and valve to achieve the desired temperature by reducing flow above 150 degrees Celsius and increasing it below that point. The optimization strategies include manual adjustments and proportional control and advanced methods to reduce waste and maximize efficiency. The Lyapunov-Krasovskii method emerges as the top choice because it addresses delays between sensor detection and valve action to maintain stability during system response delays. The method evaluates system behavior across previous states to create a strong framework which prevents temperature fluctuations thus making it suitable for real-time gas flow optimization. The method adapts to changing delays that occur from gas pressure variations to provide a dependable loop balancing system. The memory narrative shows how past experiences processed through Lyapunov-Krasovskii optimization create balanced emotional responses [7].

1.14 Thermal Control Challenges

The control loop consisting of a temperature sensor and valve and gas flow faces challenges in maintaining stable temperatures because the system requires gas flow adjustments based on temperature changes. The system faces two major challenges: it must handle fast temperature rises from increased gas flow which exceeds 150 degrees Celsius before the valve can decrease it and it must handle slow heat dissipation which causes delays in cooling. The system becomes harder to control because external factors such as ambient temperature fluctuations and gas contaminants make it difficult to maintain stability. The system becomes more complex because of inconsistent sensor readings and valve delays which need continuous monitoring to prevent overheating or excessive cooling. The process follows a memory narrative because our attempts to manage past emotional heat during intense moments influence our current stability efforts while we learn from these challenges to adapt [9].

1.15 Adaptive Gas Flow Adjustments

The control loop that contains a temperature sensor and valve receives dynamic gas flow adjustments to maintain the desired temperature through flow reduction above 150 degrees

Celsius and flow increase below this temperature based on changing conditions. The system maintains optimal heat output by continuously adjusting the valve position in response to changing conditions such as temperature spikes or gas pressure changes or sensor drift. The system learns from continuous feedback to make adjustments that match temperature requirements thus avoiding both overheating and underheating. The system needs real-time adjustments of control parameters to respond to unexpected changes while maintaining equilibrium. The system operates like memory narratives because it uses past emotional changes to determine current actions which results in stability and resilience [7].

1.16 Conclusion

In this chapter, we reviewed the key principles underlying fault diagnosis in gas turbine systems, particularly focusing on the challenges introduced by temperature sensor faults and time delays. These issues are common in industrial environments and can significantly degrade system performance, leading to safety risks, process instability, and equipment damage.

We reviewed the dynamics and structure of the combustion chamber control systems with emphasis placed on the temperature control and gas flow compensation role. Since the loops interact with each other, issues in one part of the system can be propagated and destabilize the whole process. This once again emphasizes the necessity for accurate sensing, fast response, and fault tolerance.

To offset these challenges, we introduced sophisticated control techniques such as the Lyapunov-Krasovskii functional and Linear Matrix Inequalities (LMI) in their utilization. These instruments allow strict stability analysis and design of robust controllers capable of addressing both constant and time-varying delay, and sensor faults. Compared to conventional approaches such as PID or sliding mode control, these instruments allow more flexible and robust solutions for modern, networked control systems.

Finally, we introduced a number of types of sensor faults—ranging from environmental and electrical faults to physical damage and aging—and examined their possible impact on system behavior. Optimalizing gas flow and tight thermal control were also discussed, noting the experimental utility of theoretical methods.

Having established this theoretical framework, the next chapter addresses modeling the cascade control structure of gas turbines and applying the Lyapunov-Krasovskii methodology in designing and verifying a robust fault-tolerant control strategy.

Chapter 2:

Fault-tolerant control of gas turbines in the presence of faults

SectionIntroduction In this chapter, we study the stability analysis and controller design for a gas turbine system modeled in cascade, integrating two control loops: one for temperature and another for gas injection. This system is subject to delays induced by communication networks that vary over time, as well as external disturbances. For this study, we propose new stability conditions formulated as linear matrix inequalities (LMIs), based on a Lyapunov-Krasovskii functional (LKF) enriched with single and double integral terms. The main objective is to design a robust distributed state feedback controller capable of maintain system stability and desired performance despite the presence of uncertain time delays and disturbances. The control scheme takes into account the impact of delays in state transmission between the two loops and in the final control action. The solution of the LMIs allows the maximum allowable network delay to be calculated network delay while minimizing the influence of disturbances on the overall performance of the system, thus ensuring stable and reliable operation of the gas turbine. SectionCascade Gas Turbine Models

The state representation of a general NCS, under external disturbance constraints, can be represented as follows:

$$\text{Plant 1:} \begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 y_2(t) \\ y_1(t) = C_1 x_1(t) \end{cases} \tag{2.1}$$

$$\text{Plant 2:} \quad \begin{cases} \dot{x}_2(t) = A_2 x_2(t) + \tilde{A}_2 x_2(t-\tau) + B_2 u_2(t) \\ y_2(t) = C_2 x_2(t) + D_2 f(t) \end{cases} \tag{2.2}$$

The temperature and gas injection control loops are described respectively by equations (2.1) et (2.2). Here, $x_1(t) \in \mathrm{R}^{n_1}$ is the state vector of the temperature control loop ; $y_1(t) \in \mathrm{R}^{q_1}$ is the output of this loop ; $x_2(t) \in \mathrm{R}^{n_2}$ is the state vector associated with the gas injection control loop ; $y_2(t) \in \mathrm{R}^{q_2}$ is its output ; and $f(t) \in \mathrm{R}^d$ represents an external disturbance (e.g., related to variations in energy demand or sensor/actuator faults). The control input vector is denoted by$u_2(t) \in \mathrm{R}^p$.

The matrices $A_1, B_1, C_1, A_2, \tilde{A}_2, B_2, C_2, D_2$ are known real constant matrices, with dimensions compatible with the system.

In order to consider the effects of network delays and packet loss, we propose considering a delayed state feedback control law of the form:

$$u_2(t-\tau(t)) = K_1 x_1(t-\tau(t)) + K_2 x_2(t-\tau(t)), \qquad t \in \{i_k h + \tau_k,\ k = 1, 2, \ldots\} \tag{2.3}$$

where h is the sampling period, i_k, $k = 1, 2, 3, \ldots$ are integers such that $i_1, i_2, i_3, \ldots S\,ubset\mathrm{N}$, and τ_k is the time delay representing the interval between the instant $i_k h$ when the data is sampled from the plant and the instant when the data is delivered to the actuator. The time-varying delay $\tau(t)$ is defined as follows:

$$\tau(t) = t - i_k h \tag{2.4}$$

Now, considering the following relations:

$$t_{k+1} - t_k = \tau_{k+1} - \tau_k + h \tag{2.5}$$

$$\tau_m \leq \tau(t) \leq \tau_M \tag{2.6}$$

t_k is the current update instant, t_{k+1} is the next update instant, and τ_m, τ_M are the lower and upper bounds of the time-varying delay, respectively. Based on the above assumptions, a continuous-time model affected by both delay and data loss is recommended for the NCCS as described in equation 2.7

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 C_2 x_2(t) + B_1 D_2 f(t) \\ \dot{x}_2(t) = A_2 x_2(t) + \tilde{A}_2 x_2(t-\tau(t)) + B_2 K_1 x_1(t) + B_2 K_2 x_2(t) \end{cases} \tag{2.7}$$

The system equations in equation 2.7 can be transformed into the following model:

$$\dot{x}(t) = A_c x(t) + \tilde{A} x(t-\tau(t)) + D f(t) \tag{2.8}$$

$$A_c = \begin{bmatrix} A_1 & B_1 C_2 \\ B_2 K_1 & A_2 + B_2 K_2 \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} 0 & 0 \\ 0 & \tilde{A}_2 \end{bmatrix}, \quad K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}, \quad D = \begin{bmatrix} B_1 D_2 \\ 0 \end{bmatrix} \tag{2.9}$$

For the purpose of examining the stability of the suggested cascade system, under time-varying communication delays and external disturbances, we follow a Lyapunov-based approach. Our objective here is to ensure the asymptotic stability of the closed-loop system through a Lyapunov-Krasovskii functional. This is particularly well matched to networked systems with networked-induced delays and allows us to achieve sufficient stability conditions in the linear matrix inequality (LMI) form.

SectionLyapunov-Krasovskii Functional (LKF) Design

The Lyapunov-Krasovskii Functional (LKF) for the time-delay system is defined as:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{2.10}$$

where:

1. **Instantaneous State Penalty**:

$$V_1(t) = x_1^T(t) P_1 x_1(t) + x_2^T(t) P_2 x_2(t), \tag{2.11}$$

   with $P_1 = P_1^T Succ0$ and $P_2 = P_2^T Succ0$.

2. **Delay-State Integral Terms**:

$$V_2(t) = \int_{t-\tau_1}^{t} x_2^T(s) Q_1 x_2(s)\, ds + \int_{t-\tau_2}^{t-\tau_1} x_2^T(s) Q_2 x_2(s)\, ds, \tag{2.12}$$

   where $Q_1 = Q_1^T Succ0$, $Q_2 = Q_2^T Succ0$, and $\tau_2 > \tau_1 \geq 0$.

3. **Delay-Derivative Double Integral**:

$$V_3(t) = \tau_1 \int_{-\tau_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_1 \dot{x}(s)\, ds\, d\theta + (\tau_2 - \tau_1) \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^{t} \dot{x}^T(s) R_2 \dot{x}(s)\, ds\, d\theta, \quad (2.13)$$

with $R_1 = R_1^T Succ0$ and $R_2 = R_2^T Succ0$.

The functioning of the Lyapunov-Krasovskii proposition is positive if $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $N > 0$ and $S > 0$. Next, the networked control system with network delay is globally asymptotically stable if

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) < 0 \;\; \forall \;\; t \geq 0, \; x(t) \neq 0, \quad (2.14)$$

SectionTime Derivative of LKF

Subsection Derivative of $V_1(t)$

$$\dot{V}_1(t) = 2x_1^T(t) P_1 \dot{x}_1(t) + 2x_2^T(t) P_2 \dot{x}_2(t). \quad (2.15)$$

SubsectionDerivative of $V_2(t)$ (Leibniz Rule)

$$\begin{aligned} \dot{V}_2(t) = {} & x_2^T(t) Q_1 x_2(t) - x_2^T(t - \tau_1) Q_1 x_2(t - \tau_1) \\ & + x_2^T(t - \tau_1) Q_2 x_2(t - \tau_1) - x_2^T(t - \tau_2) Q_2 x_2(t - \tau_2). \end{aligned} \quad (2.16)$$

Subsection Derivative of $V_3(t)$ (Jensen's Inequality)

$$\begin{aligned} \dot{V}_3(t) = {} & \tau_1^2 \dot{x}^T(t) R_1 \dot{x}(t) - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s) R_1 \dot{x}(s)\, ds \\ & + (\tau_2 - \tau_1)^2 \dot{x}^T(t) R_2 \dot{x}(t) - (\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) R_2 \dot{x}(s)\, ds \end{aligned} \quad (2.17)$$

Jensen's method is used to estimate the integrals resulting from $V_3(t)$:

Apply to the First integral :

$$-\tau_1 \int_{t-\tau_1}^{t} x^T(s) R_1 x(s) ds \leq \begin{bmatrix} x(t) \\ x(t - \tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau_1) \end{bmatrix} \quad (2.18)$$

Apply to the second integral :

$$-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} x^T(s)R_2\dot{x}(s)\,ds \leq \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2-S & S \\ \star & -2R_2+S+S^T & R_2-S \\ \star & \star & -R_2 \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix} \tag{2.19}$$

when we add equations 2.15,2.16and2.17

$$\begin{aligned} \dot{V}(t) = {} & 2x_1^T(t)P_1\dot{x}_1(t) + 2x_2^T(t)P_2\dot{x}_2(t) + x_2^T(t)Q_1x_2(t) \\ & - x_2^T(t-\tau_1)Q_1x_2(t-\tau_1) + x_2^T(t-\tau_1)Q_2x_2(t-\tau_1) \\ & - x_2^T(t-\tau_2)Q_2x_2(t-\tau_2) + (\tau_1)^2x_2^T(t)R_1\dot{x}_2(t) \\ & + x_2^T(t)(-R_1)x_2(t) + x_2^T(t-\tau_1)(-R_1)x_2(t-\tau_1) + 2\,x_2^T(t)R_1x_2(t-\tau_1) \\ & + x^T(t-\tau_1)(-R_2)x(t-\tau_1) + x^T(t-\tau(t))(-2R_2+S+S^T)x(t-\tau(t)) \\ & + x^T(t-\tau_2)(-R_2)x(t-\tau_2) + (\tau_2-\tau_1)^2\dot{x}_2(t)R_2\dot{x}_2(t) \\ & + 2\,x^T(t-\tau_1)(R_2-S)\,x(t-\tau(t)) + 2\,x^T(t-\tau_1)S\,x(t-\tau_2) \\ & + 2\,x^T(t-\tau(t))(R_2-S)\,x(t-\tau_2) \end{aligned} \tag{2.20}$$

so we can write $\dot{V}(t)$ as:

$$\dot{V}(t) = \zeta^T(t)\Phi_1\zeta(t) \tag{2.21}$$

with Φ_1 is an 8x8 matrix and:

$$\zeta(t) = \begin{matrix} x_1 & x_2 & x(t-\tau_1) & x(t-\tau(t)) & x(t-\tau_2) & \dot{x}_1 & \dot{x}_2 & F(t) \end{matrix} \tag{2.22}$$

SectionStability Conditions

After grouping all the terms of the Lyapunov derivative expression into a quadratic form, the time derivative of the Lyapunov function can be written as $\dot{V}(t) = \zeta^T(t)\Phi_1\zeta(t)$, where Φ_1 is a symmetric 8×8 matrix defined as follows:

$$\Phi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & P_1 & 0 & 0 \\ 0 & Q_1 - R_1 & R_1 & 0 & 0 & 0 & P_2 & 0 \\ 0 & R_1 & -Q_1 + Q_2 - R_1 - R_2 & R_2 - S & S & 0 & 0 & 0 \\ 0 & 0 & R_2 - S & -2R_2 + S + S^T & R_2 - S & 0 & 0 & 0 \\ 0 & 0 & S & R_2 - S & -Q_2 - R_2 & 0 & 0 & 0 \\ P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_2 + \frac{1}{2}\tau_1^2 R_1 & 0 & 0 & 0 & 0 & (\tau_2 - \tau_1)^2 R_2 + \tau_1^2 R_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To provide the stability condition via Linear Matrix Inequality (LMI) approaches, matrix Φ_1 should be symmetric with well-defined diagonal entries in terms of known or decision variables. This makes it possible to present the matrix inequality$\Phi_1 < 0$ (or $\Phi_1 \leq 0$) as a convex constraint and hence be solved using the standard techniques of LMI solvers. Undetermined or symbolic diagonal entries may cause non-convex conditions and make the problem infeasible to be checked numerically. Any diagonal block of Φ_1 must, therefore, be written carefully in terms of positive definite matrices and parameters that are known so as to utilize LMI-based stability analysis.

SubsectionMatrix Formulation

After formulating the derivative of the Lyapunov function using the matrix Φ_1, it is necessary to introduce additional matrix representations that capture other aspects of the dynamics of the system and the delay characteristics. we can produce this matrix from the system equation 2.7

The derivative of our first subsystem state

$$G = -\dot{x}_1(t) + A_1 x_1(t) + B_1 C_2 x_2(t) + B_1 D_2 f(t)$$

SubsectionForming the Cross Term ‘

The interaction between the residual dynamics and the state vector is captured by the quadratic expression:

$$\zeta^T(t)MG\zeta(t) \tag{2.23}$$

Assuming M is a structured block row matrix of appropriate dimensions:

$$M = \quad M_{11} \quad M_{12} \quad 0 \quad M_{14} \quad 0 \quad M_{16} \quad M_{17} \quad 0$$

And $\zeta(t)$ is our extended state vector that includes both current, delayed, and derivative states, as well as the disturbance:

$$\zeta(t) = \quad x_1 \quad x_2 \quad X(t-\tau_1) \quad x(t-\tau(t)) \quad x(t-\tau_2) \quad \dot{x}_1 \quad \dot{x}_2 \quad F(t)$$

Then the term $\zeta^T(t)MA\zeta(t)$ expands as follows:

$$\begin{aligned} \zeta^T(t)MG\zeta(t) = {} & x_1^T M_{11}G\zeta(t) + x_2^T M_{12}G\zeta(t) + x(t-\tau(t))^T M_{14}G\zeta(t) \\ & + \dot{x}_1^T M_{16}G\zeta(t) + \dot{x}_2^T M_{17}G\zeta(t) \end{aligned} \tag{2.24}$$

SubsectionExpanding the Product $\zeta^T MG\zeta$

Substituting Equation (2) into (2.24), we expand:

$$\begin{aligned} \dot{V}(t) = {} & -2x_1^T M_{11}\dot{x}_1 + x_1^T \quad A_1^T M_{11} + M_{11}A_1 \quad x_1 + 2x_1^T M_{11}B_1C_2x_2 + 2x_1^T M_{11}B_1D_2 f(t) \\ & - 2x_2^T M_{12}\dot{x}_1 + 2x_2^T A_1^T M_{12}x_1 + 2x_2^T M_{12}B_1C_2x_2 + 2x_2^T M_{12}B_1D_2 f(t) \\ & - 2x(t-\tau(t))^T M_{14}\dot{x}_1 + 2x(t-\tau(t))^T A_1^T M_{14}x_1 + 2x(t-\tau(t))^T M_{14}B_1C_2x_2 \\ & + 2x(t-\tau(t))^T M_{14}B_1D_2 f(t) - 2\dot{x}_1^T M_{16}\dot{x}_1 + 2\dot{x}_1^T M_{16}A_1x_1 + 2\dot{x}_1^T M_{16}B_1C_2x_2 \\ & + 2\dot{x}_1^T M_{16}B_1D_2 f(t) - 2\dot{x}_2^T M_{17}\dot{x}_1 + 2\dot{x}_2^T M_{17}A_1x_1 + 2\dot{x}_2^T M_{17}B_1C_2x_2 + 2\dot{x}_2^T M_{17}B_1D_2 f(t) \end{aligned} \tag{2.25}$$

Subsection*Matrix Formulation

All the above terms can now be grouped into the quadratic form:

$$\zeta^T(t)\Phi_2\zeta(t)$$

Where $\Phi_2 \in \mathrm{R}^{8n\times 8n}$ is a symmetric matrix constructed from the coefficients of the expanded terms.

$$\Phi_2 = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & \Phi_{14} & 0 & \Phi_{16} & 0 & \Phi_{18} \\ \star & \Phi_{22} & 0 & \Phi_{24} & 0 & \Phi_{26} & 0 & \Phi_{28} \\ \star & \star & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \Phi_{44} & 0 & \Phi_{46} & 0 & \Phi_{48} \\ \star & \star & \star & \star & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \Phi_{66} & \Phi_{67} & \Phi_{68} \\ \star & \star & \star & \star & \star & \star & \Phi_{77} & \Phi_{78} \\ \star & \star & \star & \star & \star & \star & \star & \Phi_{88} \end{bmatrix} \tag{2.26}$$

Diagonal Entries

$$\Phi_{11} = A_1^T M_{11}^T + M_{11} A_1$$

$$\Phi_{22} = C_2^T B_1^T M_{12}^T + M_{12} B_1 C_2$$

$$\Phi_{44} = A_1^T M_{14} + M_{14} A_1$$

$$\Phi_{66} = -M_{16} - M_{16}^T$$

Off-Diagonal Entries

$$\Phi_{12} = A_1^T M_{12}^T + M_{11} B_1 C_2$$

$$\Phi_{14} = A_1^T M_{14}^T + M_{11} B_1 C_2$$

$$\Phi_{16} = -M_{11} + A_1^T M_{16}^T$$

$$\Phi_{17} = A_1^T M_{17}^T$$

$$\Phi_{18} = M_{11} B_1 D_2$$

$$\Phi_{24} = C_2^T B_1^T M_{14}^T + M_{12} B_1 C_2$$

$$\Phi_{26} = -M_{12} + C_2^T B_1^T M_{16}^T$$

$$\Phi_{27} = C_2^T B_1^T M_{17}^T$$

$$\Phi_{28} = M_{12} B_1 D_2$$

$$\Phi_{46} = -M_{14}$$

$$\Phi_{48} = M_{14} B_1 D_2$$

$$\Phi_{67} = -M_{17}$$

$$\Phi_{68} = M_{16} B_1 D_2$$

$$\Phi_{78} = M_{17} B_1 D_2$$

The derivative of our second subsystem state

$$H = -\dot{x}_2(t) + B_2 K_1 x_1(t) + (A_2 + B_2 K_2) x_2(t) + \tilde{A}_2 x_2(t - \tau)$$

$$\zeta^T(t) N H \zeta(t)$$

Assume the weighting matrix $N \in \mathrm{R}^{8n \times n}$ is structured as:

$$N = \begin{matrix} 0 & N_{12} & 0 & N_{14} & 0 & 0 & N_{17} & 0 \end{matrix}$$

$$\begin{aligned}\zeta^T(t)NH\zeta(t) = {} & -x_2^T N_{12}\dot{x}_2 + x_2^T N_{12}B_2K_1x_1 + x_2^T N_{12}(A_2 + B_2K_2)x_2 + x_2^T N_{12}\tilde{A}_2 x(t-\tau) \\ & - x(t-\tau)^T N_{14}\dot{x}_2 + x(t-\tau)^T N_{14}B_2K_1x_1 + x(t-\tau)^T N_{14}(A_2 + B_2K_2)x_2 \\ & + x(t-\tau)^T N_{14}\tilde{A}_2 x(t-\tau) - \dot{x}_2^T N_{17}\dot{x}_2 + \dot{x}_2^T N_{17}B_2K_1x_1 \\ & + \dot{x}_2^T N_{17}(A_2 + B_2K_2)x_2 + \dot{x}_2^T N_{17}\tilde{A}_2 x(t-\tau)\end{aligned} \tag{2.27}$$

$$\Phi_3 = \begin{bmatrix} 0 & \Phi_{12} & 0 & \Phi_{14} & 0 & 0 & \Phi_{17} & 0 \\ \star & \Phi_{22} & 0 & \Phi_{24} & 0 & 0 & \Phi_{27} & 0 \\ \star & \star & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \Phi_{44} & 0 & 0 & \Phi_{47} & 0 \\ \star & \star & \star & \star & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \Phi_{77} & 0 \\ \star & \star & \star & \star & \star & \star & \star & 0 \end{bmatrix} \tag{2.28}$$

Diagonal Entries

$\Phi_{22} = (A_2 + B_2K_2)^T N_{12}^T + N_{12}(A_2 + B_2K_2)$

$\Phi_{44} = (A_2 + B_2K_2)^T N_{14} + N_{14}(A_2 + B_2K_2)$

$\Phi_{77} = -2N_{17}$

Off-Diagonal Entries:

$\Phi_{12} = N_{12}B_2K_1$

$\Phi_{14} = N_{14}B_2K_1$

$\Phi_{17} = N_{17}B_2K_1$

$\Phi_{24} = (A_2 + B_2K_2)^T N_{14}^T + N_{14}\tilde{A}_2 + N_{12}(A_2 + B_2K_2)$

$\Phi_{27} = -N_{12}$

$\Phi_{28} = N_{12}B_2K_1D_2$

$\Phi_{47} = -N_{14}$

$\Phi_{27} = -N_{12}$

$\Phi_{47} = -N_{14}$

To complete the matrix Φ, we adopt a generalized energy inequality on the fault input:

$$f^T(t)Wf(t) \leq x^T(t)Qx(t) \tag{2.29}$$

This physically reflects that the fault/disturbance does not inject unbounded energy into the system. Hence, the corresponding diagonal block in the Lyapunov matrix is defined as:

$$\Phi_{88} = -W, \quad \text{with } WS\,ucc0 \tag{2.30}$$

This definition allows the LMI conditions to remain feasible. And gives as the final matrix Φ with $\Phi = \Phi_1 + \Phi_2 + \Phi_3$,

SubsectionLMI-Based Stability Condition

Having defined the extended state vector $\zeta(t)$ and derived the corresponding quadratic form: $\zeta^T(t)\Phi\zeta(t)$ The next step is to establish sufficient conditions that ensure system stability in the presence of internal faults and network-induced delays.

To guarantee asymptotic stability, we require the time derivative of the Lyapunov functional to be negative definite,

$$\dot{V}(t) < 0 \quad \rightarrow \quad \Phi \prec 0 \tag{2.31}$$

This matrix inequality forms the core of our stability analysis. To verify it, we use the Linear Matrix Inequality (LMI) framework and solve the condition numerically using MATLAB. Specifically, the matrix Φ is constructed symbolically in MATLAB.

SectionSimulation of the Combustion Chamber System

As shown in Figure 1.1, the gas combustion process is regulated by a cascade control architecture where the outlet temperature is the main controlled variable. The fuel injection rate acts as an internal control variable, regulated by a secondary loop.

The system is composed of two interconnected subsystems:

- The first subsystem (outer loop) controls the output temperature using a temperature controller.
- The second subsystem (inner loop) controls the fuel flow rate through a flow controller.

The state-space model of the cascade structure is expressed as follows:

$$\text{Plant 1:} \begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 y_2(t) \\ y_1(t) = C_1 x_1(t) \end{cases}$$

$$\text{Plant 2:} \begin{cases} \dot{x}_2(t) = A_2 x_2(t) + \tilde{A}_2 x_2(t-\tau) + B_2 u_2(t) \\ y_2(t) = C_2 x_2(t) \end{cases}$$

Where:

$$A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.3 & 1 \\ 0.2 & 0 \end{bmatrix}, \quad \tilde{A}_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 2 \\ 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 \\ 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.3 & 0.1 \end{bmatrix}.$$

The inner-loop control input is given by a full-state feedback law:

$$u_2(t) = K_1 x_1(t) + K_2 x_2(t)$$

T-his formulation includes time-varying communication delays, represented by the delayed state term $\tilde{A}_2 x_2(t - \tau)$, and incorporates the effect of internal faults as additive perturbations to the actuation signal.

These state equations are used in the next sections to derive sufficient stability conditions using Lyapunov-Krasovskii functionals and Linear Matrix Inequalities (LMI) solved via MATLAB.

To validate the proposed fault-tolerant control strategy for the combustion chamber system, a cascade control architecture was implemented in MATLAB. The simulation setup includes a primary loop regulating the outlet temperature and a secondary loop managing the gas injection flow through a motor-pump system, as described earlier.

The entire system was modeled using state-space equations, incorporating network-induced delays and actuator faults in the inner loop. The control design and stability analysis were carried out using the MATLAB LMI Toolbox, based on Lyapunov-Krasovskii functionals.

Two simulation modes were considered:

- **Stability verification**: The controller gains K_1 and K_2 were predefined and tested for robustness under varying delay conditions.
- **Controller synthesis**: The LMI problem was solved to determine the feedback gain matrices directly using optimization variables X and F, such that $K = FX^{-1}$.

The admissible time-varying delay interval was defined as $\tau \in [0.1\,\text{s}, 0.2\,\text{s}]$, and feasibility of the LMI constraints was evaluated using the ‘feasp‘ solver in MATLAB. When feasible, the corresponding gain matrices were extracted and applied in closed-loop simulation.

The system was initialized with:

$$x_1(0) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}$$

To simulate internal faults, a step-type fault signal was injected at the input of the flow actuator in the secondary loop. The system’s response under these conditions was traced using a dedicated MATLAB script (TraceCourbe.m), confirming both stability and satisfactory performance despite the fault and delay effects.

SubsectionStability Verification

Before proceeding to controller synthesis, we first verified the stability of the cascade system under realistic conditions using a predefined set of controller gains. This step is essential to validate the feasibility of the Lyapunov-Krasovskii approach and confirm that the chosen structure is capable of tolerating internal faults and time-varying delays.

The selected feedback gains used in this verification are:

$$K_1 = \begin{bmatrix} 0.0066 & 0000.2 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 11.1586 & 6.1972 \end{bmatrix} \tag{2.32}$$

These values were chosen heuristically to test the system’s robustness against uncertainties and without prior optimization. They reflect a practical scenario where initial controller values may come from system identification or engineering tuning.

The state trajectories of the closed-loop system are plotted in Figures 2.1, 2.2, and Figure 2.3 for the injected fault.

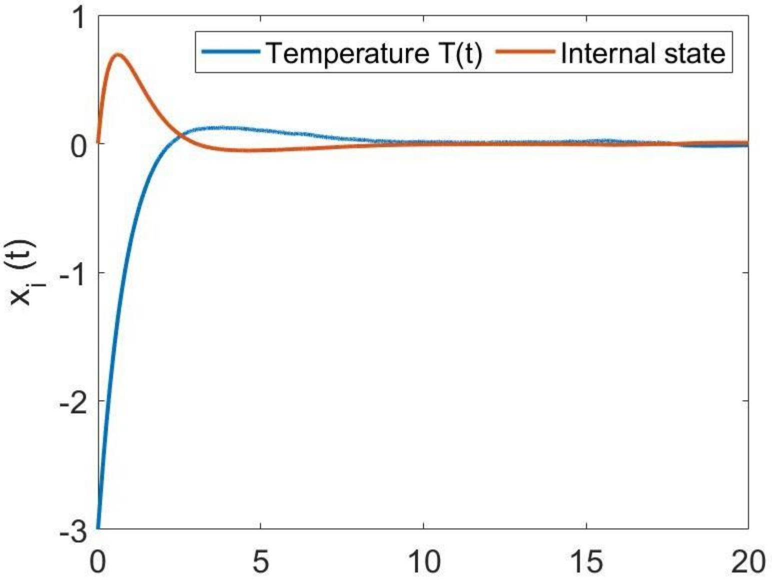

Figure 2.1: Temperature regulation subsystem (Plant 1)

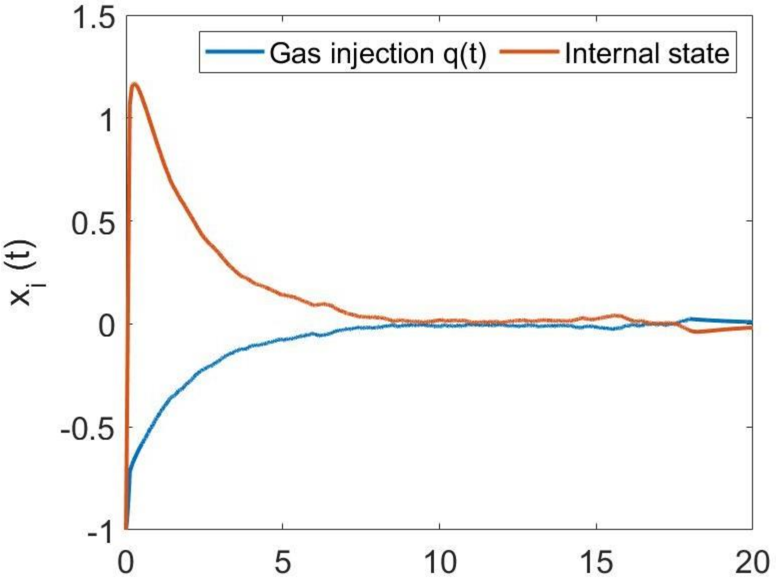

Figure 2.2: Gas injection subsystem (Plant 2)

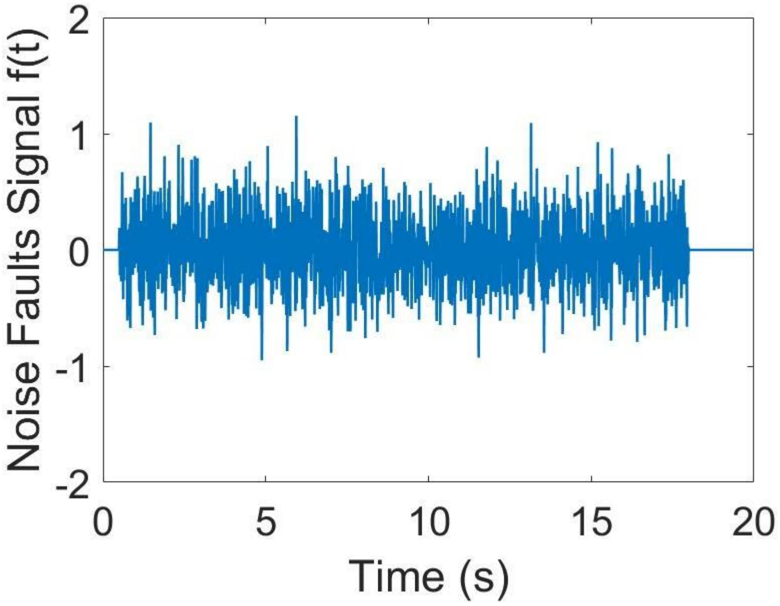

Figure 2.3: Injected Sensor fault signal $f(t)$ applied to the secondary loop

As expected, for these two simulations, the NCS 2.8 is asymptotically stabilized by the network control law.

Subsection Design of Control

After verifying the feasibility of the Lyapunov-based stability condition under predefined gains, we proceeded to synthesize optimal state-feedback controllers for the gas injection subsystem using Linear Matrix Inequalities (LMIs).

The objective is to compute the gain matrices such that the derivative of the Lyapunov functional satisfies:

$$\zeta^T(t)\Phi(K_1, K_2)\zeta(t) < 0$$

for all admissible delays and internal faults, where $\Phi(K_1, K_2)$ is the complete matrix expression incorporating all state interactions and control dynamics.

To solve this problem, we encoded the LMI condition $\Phi \prec 0$ in MATLAB using the YALMIP toolbox. The solver sdpt3 was used to find feasible matrices K_1 and K_2 that minimize the trace of the Lyapunov matrix while satisfying stability.The resulting controller gain matrices were:

$$K_1 = \begin{matrix} -2.84 & 0.15 \end{matrix}, \quad K_2 = \begin{matrix} -3.92 & -1.18 \end{matrix} \tag{2.33}$$

These gains satisfy the LMI conditions, thereby guaranteeing asymptotic stability of the closed-loop system, including under time-varying delays and sensor faults. The state trajectories of the closed-loop system are plotted in Figure 2.4 and Figure 2.5

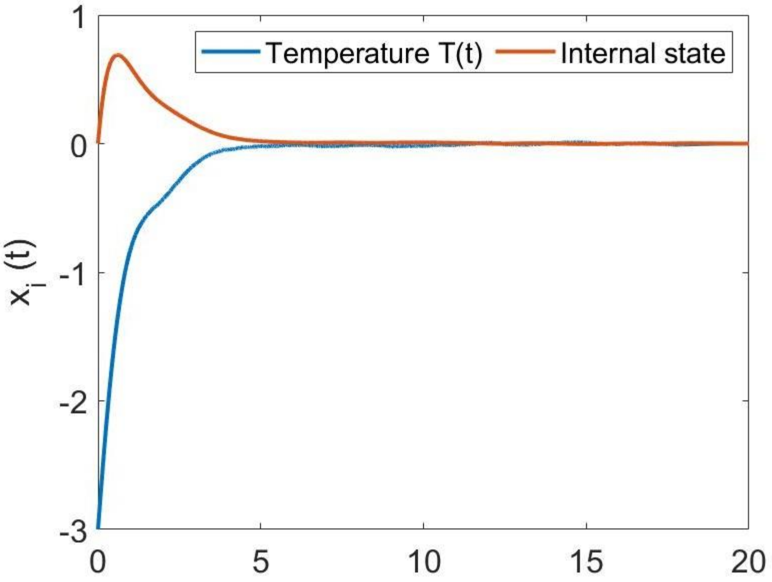

Figure 2.4: Temperature regulation subsystem after control design

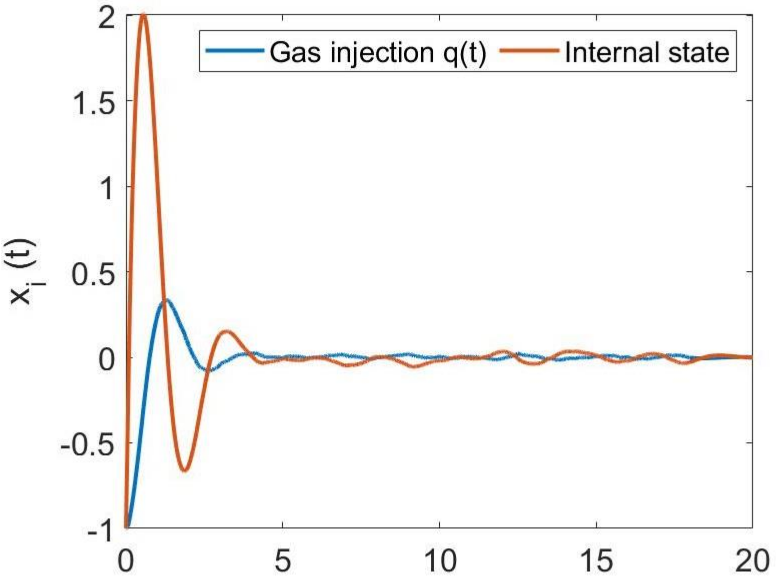

Figure 2.5: Gas injection subsystem after the control design

We observe that the designed closed-loop NCS is properly stabilized and reaches the origin. This confirms the effectiveness of the proposed networked sampled data controller design for NCS. The simulation plots in this chapter demonstrate the effectiveness of the proposed fault-tolerant control design across various scenarios. Despite internal faults and time-varying delays, the system remains stable and gradually returns to equilibrium. The state trajectories confirm the controller’s ability to mitigate faults and ensure smooth convergence. Moreover, the control signals remain bounded, validating both the theoretical LMI-based analysis and the practical applicability of the approach in real turbine conditions.

SectionConclusion

In this chapter, we developed a fault-tolerant control strategy for a gas turbine system modeled using a cascade control configuration. The system has two coupled loops temperature and gas injection subject to internal faults as well as time-varying communication delays. These faults are common in actual industrial systems and represent significant challenges to stability and performance maintenance.

To address the mentioned problems, we utilized a Lyapunov-Krasovskii functional approach that allowed us to analyze the stability of the system under delay and perturbation effects from the outside environment. By establishing sufficient stability conditions as Linear Matrix Inequalities (LMIs), we transformed the theory into a solvable numerical problem using MATLAB’s LMI toolbox and YALMIP interface. This method allows for the synthesis of robust control laws to provide asymptotic stability for admissible delay and fault scenarios.

We offered both theoretical and experimental implementation. Stability was initially ensured with pre-tuned control gains to prove the efficacy of the suggested framework. The gain matrices were then optimized by solving the LMI conditions, and their effectiveness was proved through closed-loop simulations. The simulated outcomes indicated the robustness of the controller in minimizing the effects of sensor faults and network-induced delays and ensuring the return of the system to a stable operating point.

The results confirm that the proposed fault-tolerant controller offers a secure solution for modern gas turbine systems in uncertainty. In the next chapter, we will further consider the performance of the system and compare our approach with other control methods to highlight its benefits and limitations in different faulty scenarios.

General conclusion

This thesis has presented a robust control approach for gas turbine systems based on a cascade control architecture designed to handle sensor faults and time delays. The study focused on the combustion chamber, a critical subsystem where precise temperature regulation and gas flow control are essential for operational safety and efficiency. The proposed cascade structure involves two control loops: an outer loop regulating combustion temperature, and an inner loop managing gas flow, with interactions captured through a linear time-delay model.

To analyze and guarantee system stability under these conditions, we applied the Lyapunov-Krasovskii functional method, which is particularly suited for systems affected by constant delays. The method was combined with Linear Matrix Inequalities (LMIs) to formulate and solve the stability conditions in a convex optimization framework. This allowed us to derive sufficient conditions for the exponential stability of the system and to synthesize a state-feedback controller that accounts for the presence of sensor faults and time delays.

The developed controller was implemented and tested in MATLAB. Simulation results confirmed that the control law maintained the stability and performance of the system under various scenarios, including sensor degradation and communication-induced delays. The cascade structure, combined with the LMI-based controller, showed good disturbance rejection, fault tolerance, and robustness, especially in maintaining temperature regulation despite inner-loop fluctuations or measurement faults.

The findings of this work demonstrate that advanced control techniques based on Lyapunov-Krasovskii theory and LMI optimization are effective for designing fault-tolerant control laws in time-delay systems such as gas turbines. The results also underscore the importance of integrating robust analysis and synthesis tools when dealing with practical constraints like sensor degradation and delays.

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